$\|u\|^2 t^2 + 2t\langle u,v\rangle + \|v\|^2 \geq 0$ $[\because \|tu+v\|^2 \geq 0]$

$at^2+bt+c \geq 0 \quad \forall t$, $a=\|u\|^2$, $b=2\langle u,v\rangle$, $c=\|v\|^2$

$\therefore b^2-4ac \leq 0$

$(2\langle u,v\rangle)^2 - 4\|u\|^2\|v\|^2 \leq 0$

$\Rightarrow 4\langle u,v\rangle^2 \leq 4\|u\|^2\|v\|^2$

$\Rightarrow |\langle u,v\rangle| \leq \|u\|\|v\|$ □

* $(a_1,a_2,\ldots,a_n) \in \mathbb{R}^n$ or $\mathbb{C}^n$

$\|(a_1,a_2,\ldots,a_n)\|_\infty = \max(|\{a_i\}|)$

$\|1,2,\ldots,10\|_\infty = \max(|1|,|2|,\ldots,|10|) = 10$

$\|(a_1,a_2,\ldots,a_n)\|_1 = \sum_{i=1}^{n}|a_i|$

$\|(a_1,a_2,\ldots,a_n)\|_2 = \sqrt{|a_1|^2+|a_2|^2+\cdots+|a_n|^2}$

Ex:- $u=(1,2,3,-1)$

Find $\|u\|_\infty$, $\|u\|_1$, $\|u\|_2$

A:- $\|u\|_\infty = \max\{|1|,|2|,|3|,|-1|\}$

$= \max\{1,2,3,1\} = 3$

$\|u\|_1 = |1|+|2|+|3|+|-1| = 7$

$\|u\|_2 = \sqrt{|1|^2+|2|^2+|3|^2+|-1|^2} = \sqrt{15}$

P Kalika Maths