

Ex:-3 $P_n(\mathbb{R}) =$ Set of all polynomials of degree $\leq n$.

$$\mathbb{F} = \mathbb{R}$$

Let $p(x), q(x) \in P_n(\mathbb{R})$

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

$$q(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + b_0$$

$P_n(\mathbb{R})$ is a v.s.

Ex:-4 $S = \{ \text{The set of polynomials of degree } n \}$, $\mathbb{F} = \mathbb{R}$

Not satisfying closure property.

So $S(\mathbb{R})$ is not a vector space.

$$\text{Let } p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

$$q(x) = -a_n x^n + b_{n-1} x^{n-1} + \dots + b_0$$

$$p+q = c_n x^{n-1} + \dots + c_0 \notin S.$$

Ex:-5 $\mathbb{R}^+(\mathbb{R})$

$$u+v = u \cdot v$$

$$u \cdot u = u^2$$

$$0+v = 0 \cdot v = 0 \notin \mathbb{R}^+$$

$$(1) u \in \mathbb{R}^+$$

$$v \in \mathbb{R}^+$$

$$u+v = u \cdot v \in \mathbb{R}^+$$

$$(2) (u+v) + w = uvw$$

$$u + (v+w) = uvw$$

$$(3) u \in \mathbb{R}^+$$

$$1 \in \mathbb{R}^+$$

$$u+1 = u \cdot 1 = u$$

$$(4) u \in \mathbb{R}^+$$

$$\frac{1}{u} \in \mathbb{R}^+$$