Ex:- Check LI or LD

$S = \{(1,1,1), (1,1,-1), (1,2,3)\}$

Put $\alpha_1(1,1,1) + \alpha_2(1,1,-1) + \alpha_3(1,2,3) = 0$

$\Rightarrow (\alpha_1+\alpha_2+\alpha_3, \alpha_1+\alpha_2+2\alpha_3, \alpha_1-\alpha_2+3\alpha_3) = 0$

$\Rightarrow \alpha_1+\alpha_2+\alpha_3 = 0$ —— (i)

$\alpha_1+\alpha_2+2\alpha_3 = 0$ —— (ii)

$\alpha_1 - \alpha_2 + 3\alpha_3 = 0$ —— (iii)

Adding (i) & (iii) $2\alpha_1 + 4\alpha_3 = 0$

$\Rightarrow \alpha_1 = -2\alpha_3$

$\alpha_1+\alpha_2+2\alpha_3 = 0$

$\Rightarrow \alpha_2 = 0$

Subtracting (i) & (iii)

$2\alpha_3 = 0$

$\Rightarrow \alpha_3 = 0$

~~$\alpha_1 = -\alpha_2$~~

$(\alpha_1, \alpha_2, \alpha_3) = (-2, 0, 1)\alpha_3$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & -2 & 2 \end{pmatrix}$$

rank = 3 = no. of vectors.

$\therefore$ S is LI.

Span of S:-

Let S be any non-empty subset of a v.s V, then the span of S is the set of all finite linear combinations of S. and it is denoted by [S].

* Prove that [S] is a subspace of V.

Proof:- Let $S = \{u_1, u_2, \dots, u_n\}$

$[S] = [u_1, u_2, \dots, u_n]$

let $u = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n$

$v = \beta_1 u_1 + \beta_2 u_2 + \dots + \beta_n u_n$