Dimension of symmetric matrix $= n + \frac{n^2-n}{2}$

$$= \frac{2n+n^2-n}{2} = \frac{n^2+n}{2} = \frac{n(n+1)}{2}$$

* Find the dimension of skew-symmetric matrix.

Q:- Check that $B = \{(1,1,1), (1,2,3), (1,0,0)\}$ is a basis or not.

A:- Now,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 0 \end{vmatrix} = 3-2 = 1 \neq 0.$$

$\therefore$ B is LI.

Let $(x,y,z) \in \mathbb{R}^3$ and

$$(x,y,z) = \alpha(1,1,1) + \beta(1,2,3) + \gamma(1,0,0)$$

$$= (\alpha+\beta+\gamma, \alpha+2\beta, \alpha+3\beta)$$

Comparing both sides, we get,

$$\alpha+\beta+\gamma = x \longrightarrow \text{(i)}$$

$$\alpha+2\beta = y \longrightarrow \text{(ii)}$$

$$\alpha+3\beta = z \longrightarrow \text{(iii)}$$

eq(iii) $-$ eq(ii) $\Rightarrow \beta = z-y$

eq(i) $\Rightarrow \alpha+\beta+\gamma = x$

$\Rightarrow \alpha + z - y + \gamma = x$

$\Rightarrow \alpha + \gamma = x+y-z \longrightarrow \text{(iv)}$

eq(ii) $\Rightarrow \alpha = y - 2\beta$

$= y - 2z + 2y = 3y - 2z$