Now, let $v_1, v_2 \in W_2$ such that

$v_1 = (a_1, b_1, c_1) \in \mathbb{R}^3$, $2a_1 + c_1 = 0$

$v_2 = (a_2, b_2, c_2) \in \mathbb{R}^3$, $2a_2 + c_2 = 0$

$v_1 + v_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$, $2a_1 + c_1 + 2a_2 + c_2 = 0$

$\Rightarrow 2(a_1 + a_2) + (c_1 + c_2) = 0$

$\therefore v_1 + v_2 \in W_2$

$\alpha v_1 = \alpha(a_1, b_1, c_1) = (\alpha a_1, \alpha b_1, \alpha c_1)$, $\alpha(2a_1 + c_1) = 0$

$\Rightarrow 2(\alpha a_1) + \alpha c_1 = 0$

$\therefore \alpha v_1 \in W_2$

$\therefore W_2$ is a subspace of $\mathbb{R}^3$.

Let $u, v \in W_1 \cap W_2$

$\Rightarrow u, v \in W_1 \wedge u, v \in W_2$

$\Rightarrow \alpha u + \beta v \in W_1 \wedge \alpha u + \beta v \in W_2$ [$\because W_1$ & W_2 are subspaces of V]

$\Rightarrow \alpha u + \beta v \in W_1 \cap W_2$

$\therefore W_1 \cap W_2$ is a subspace of V.

□

Q:- Find the dimension of a skew-symmetric matrix.

A:- Let us consider a $n \times n$ skew-symmetric matrix. In this matrix all the diagonal elements are zero and $a_{ij} = -a_{ji}$, $j \neq i$

$\therefore$ Dimension $= \dfrac{n^2 - n}{2}$.