Co-ordinate of a vector :-

Let $B = \{v_1, v_2, \dots, v_n\}$ be an ordered basis of a v.s V, then any vector $v \in V$ can be written as,

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

The vector $(\alpha_1, \alpha_2, \dots, \alpha_n)$ is known as the co-ordinate vector of v relative to the ordered basis B and it is denoted by $[v]_B$.

$V = \mathbb{R}^4$

* $S = \{(1,1,1,0), (1,2,3,4), (1,1,1,-1), (0,0,0,1), (0,0,1,0), (1,-1,-2,-3)\}$

First Find the rank.

The rank must be 4.

And exclude the vector which gives zero rows.

* Casting out Algorithm

$$\begin{matrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$\begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array}$$
$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & -1 \\ 1 & 3 & 1 & 0 & 1 & -2 \\ 0 & 4 & -1 & 1 & 0 & -3 \end{pmatrix} \sim \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 0 & 1 & -3 \\ 0 & 4 & -1 & 1 & 0 & -3 \end{pmatrix} \begin{matrix} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 4R_2 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} \boxed{1} & 1 & 1 & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 0 & 0 & -2 \\ 0 & 0 & \boxed{-1} & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & \boxed{1} & 1 \end{pmatrix}$$

$\therefore$ v_4 is the linear combination of prev.(3) vectors.

$B = \{v_1, v_2, v_3, v_5\}$