We can see in eq(1) LHS is lying in U and RHS lying in W.

So we can conclude that this element is lie in $U \cap W$ from eq (A).

$$\Rightarrow -\sum_{i=r+1}^{p} r_i w_i = \sum_{i=1}^{r} \delta_i v_i$$

$$\Rightarrow \cancel{\sum_{i=1}^{r} \alpha_i v_i + \sum_{i=r+1}^{m} \beta_i u_i = \sum_{i=1}^{r} \delta_i v_i}$$

$$\Rightarrow \sum_{i=1}^{r} \delta_i v_i + \sum_{i=r+1}^{p} r_i w_i = 0$$

Since S_3 is the basis of W,

$$\Rightarrow r_i = 0, \quad i = r+1, \dots, p$$
$$\delta_i = 0, \quad i = 1, \dots, r$$

Now from eq (A).

$$\sum_{i=1}^{r} \alpha_i v_i + \sum_{i=r+1}^{m} \beta_i u_i = 0 \quad [\because r_i = 0,\ i = r+1, \dots, p]$$

$$\Rightarrow \alpha_i = 0, \quad i = 1, \dots, r$$
$$\beta_i = 0, \quad i = r+1, \dots, m$$

$\because$ S_2 is a basis of U.

$\therefore$ Set B is LI.

HW Now it remains to show that $[B] = U + W$