LINEAR TRANSFORMATION

↳ Let U and V are two vector spaces over the same field. Then the mapping $T : U \rightarrow V$ is a said to be a linear map or linear transformation / linear operator if

$$T(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2) \quad \forall \; u_1, u_2 \in U$$

& α, β are scalars.

Ex:-

$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(x, y) = (x+y, y)$

Let $u_1 = (x_1, y_1)$

$u_2 = (x_2, y_2)$

$\Rightarrow T(u_1) = (x_1 + y_1, y_1)$

$T(u_2) = (x_2 + y_2, y_2)$

$$T(\alpha u_1 + \beta u_2) = T\left(\alpha(x_1, y_1) + \beta(x_2, y_2)\right)$$
$$= T(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2)$$
$$= \left(\alpha x_1 + \beta x_2 + \alpha y_1 + \beta y_2, \alpha y_1 + \beta y_2\right)$$

$$\alpha T(u_1) + \beta T(u_2) = \alpha(x_1 + y_1, y_1) + \beta(x_2 + y_2, y_2)$$
$$= (\alpha x_1 + \alpha y_1, \alpha y_1) + (\beta x_2 + \beta y_2, \beta y_2)$$
$$= (\alpha x_1 + \alpha y_1 + \beta x_2 + \beta y_2, \alpha y_1 + \beta y_2)$$

$\therefore T(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2)$

$\therefore$ T is linear.