

Assign:-

Can we defined a linear transformation in different field.

Q:- $T: \mathbb{R}^2(\mathbb{R}) \longrightarrow \mathbb{R}^3(\mathbb{R})$

$$T(1,1) = (1, 2, 3)$$

$$T(1,-1) = (1, 0, 0) \quad , \text{ if possible find } T.$$

$$(x, y) \in \mathbb{R}^2$$

$$(x, y) = \alpha(1, 1) + \beta(1, -1)$$

$$\alpha + \beta = x$$

$$\alpha - \beta = y$$

$$\Rightarrow \alpha = \frac{x+y}{2}, \quad \beta = \frac{x-y}{2}$$

$$T(x, y) = T\left(\frac{x+y}{2}(1, 1) + \frac{x-y}{2}(1, -1)\right)$$

$$= \frac{x+y}{2} T(1, 1) + \frac{x-y}{2} T(1, -1)$$

$$= \frac{x+y}{2} (1, 2, 3) + \frac{x-y}{2} (1, 0, 0)$$

$$= \left(x, x+y, \frac{3}{2}(x+y) \right)$$

* If $T(1, 2) = (1, 0, 0)$, then T doesn't exist.

* $T: P_3 \longrightarrow P_3$

$$T(1+x) = 1+x$$

$$T(2+x) = x+3x^2$$

$$T(x^2) = 0$$

A:- $\{1+x, 2+x, x^2\}$ is L.I.