$$\begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda(6-5\lambda+\lambda^2) - 2(-2+\lambda) = 0$

$\Rightarrow -6\lambda + 5\lambda^2 - \lambda^3 + 4 - 2\lambda = 0$

$\Rightarrow \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$

$\Rightarrow \lambda^3 - \lambda^2 - 4\lambda^2 + 4\lambda + 4\lambda - 4 = 0$

$\Rightarrow \lambda^2(\lambda-1) - 4\lambda(\lambda-1) + 4(\lambda-1) = 0$

$\Rightarrow (\lambda-1)(\lambda^2 - 4\lambda + 4) = 0$

$\Rightarrow (\lambda-1)(\lambda-2)^2 = 0$

$\Rightarrow \lambda = 1, 2, 2$

For $\lambda = 1$

$AX = \lambda X$

$\Rightarrow AX = X$

$$\Rightarrow \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -2x_3 \\ x_1 + 2x_2 + x_3 \\ x_1 + 3x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -x_1 - 2x_3 \\ x_1 + x_2 + x_3 \\ x_1 + 2x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$