$$\xrightarrow{R_3 \leftrightarrow R_4} \begin{array}{c} 4 \\ 1 \\ 2 \\ 3 \\ 5 \\ 6 \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{pmatrix} \xrightarrow[R_6 \to R_6 + 3R_2]{R_5 \to R_5 + 2R_2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_5 \to R_5 - R_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_6 \to R_6 - R_4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ The set $\{(x-2), (x^2-4), (x^3-8), 1\}$ is LI.

$\therefore$ $U + W = \{ \alpha(x-2) + \beta(x^2-4) + \gamma(x^3-8) + \delta \mid \alpha, \beta, \gamma, \delta \in \mathbb{R} \}$

2) Given, U and W are two distinct subspaces of v.s V

$\dim(V) = 7$

$\dim(U) = 5$

$\dim(W) = 4$

$\therefore \dim(U \cap W) \leq 4$

$\dim(U+W) \leq \dim(V)$

$\Rightarrow \dim(U+W) \leq 7$

$\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$

$\Rightarrow \dim(U \cap W) = \dim(U) + \dim(W) - \dim(U+W)$

$\geq 5 + 4 - 7$

$= 2$